10/10 Recitation

1. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$

   (a) Show that $A\mathbf{x} = \mathbf{b}$ has no solution.

   (b) Find a least squares solution $\hat{\mathbf{x}}$ to $A\mathbf{x} = \mathbf{b}$.

   (c) How does $A\hat{\mathbf{x}}$ compare to $\mathbf{b}$, geometrically?

   (d) Recall that the $A\backslash b$ command in Julia can be used to solve the system $A\mathbf{x} = \mathbf{b}$. This command also gives the least squares solution (best approximate solution) to $A\mathbf{x} = \mathbf{b}$ if the equation has no solution. Open the **Julia9LeastSquares** notebook and work through the commands in the first section to check your answer above.

2. Consider the points $(-1, 2), (0, 0), (1, -3), (2, 5)$ in the $xy-$plane.

   (a) Sketch the points in the $xy-$plane. Notice a parabola might better fit the data than a line; sketch one.

(b) Find the parabola $y = a + bx + cx^2$ that best fits these data points.

(c) Determine the vertical distances between this parabola and the data points (i.e. find the error vector).

(d) Work through commands in the second section of the **Julia9LeastSquares** notebook to check your work above and to plot the parabola of best fit.

(e) Use Julia to find a *line* of best fit. Compare the error vector in this case to the error vector in the parabola case to determine which curve better fits the data. Recall that you can calculate the length of a vector in Julia using the "norm(v)" function.

3. A system $A\mathbf{x} = \mathbf{b}$ does not necessarily have a unique least squares solution; this occurs if A does not have linearly independent columns. Come up with an example of such a matrix A and a vector $\mathbf{b} \notin C(A)$ and find a least squares solution.